State of state Transition Matrix: It defines the transition from initial time $t=0$ to any time when i/p is zero. It is a matrix which satisfies the linear homogeneous differential eqn.

$$\dot{x}(t) = \frac{d}{dt}x(t) = Ax(t) \rightarrow (1)$$

$\phi(t)$ = state transition matrix

$$\frac{d\phi(t)}{dt} = A\phi(t) \rightarrow (2)$$

$$x(t) = \phi(t)\,x(0) \rightarrow (3)$$

Take Laplace of eqn (1)

$$sX(s) - X(0) = AX(s)$$

$$I\,sX(s) - X(0) = AX(s)$$

$$(sI - A)\,X(s) = X(0)$$

$$\therefore X(s) = X(0)\,(sI-A)^{-1} \rightarrow (4)$$

$$\therefore x(t) = \underbrace{\mathcal{L}^{-1}(sI-A)^{-1}}_{\phi(t)}\,x(0)$$

Compare eqn (3) and eqn 4.

$$\boxed{\phi(t) = \mathcal{L}^{-1}(sI-A)^{-1}}$$

State transmission of

Q. Determine state transition Matrix. (multi i/p single o/p)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$y(t) = \underbrace{\begin{bmatrix}1&1\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad I = \begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\dot{x}(t) = AX + BU$$

$$y(t) = CX.$$

$$\boxed{\phi(t) = L^{-1}\left[sI - A\right]^{-1}}$$

$$A = \begin{bmatrix}1&0\\1&1\end{bmatrix} \quad B = \begin{bmatrix}1\\1\end{bmatrix}, \quad C = \begin{bmatrix}1&1\end{bmatrix}$$

$$\left[sI - A\right] \qquad = \begin{bmatrix}s&0\\0&s\end{bmatrix} \qquad I = \begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\underline{sI-A} = \begin{bmatrix}s&0\\0&s\end{bmatrix} - \begin{bmatrix}1&0\\1&1\end{bmatrix} = \begin{bmatrix}s-1&0\\-1&s-1\end{bmatrix}$$

To get $\left[sI - A\right]^{-1} = \dfrac{\text{Adj}\left[sI-A\right]}{\text{Mod}\left[sI-A\right]}$

$$\text{Mod}\left[sI-A\right] = \begin{vmatrix} \overset{a}{s-1} & \overset{b}{0} \\ \overset{c}{-1} & \overset{d}{s-1}\end{vmatrix} = ad - bc = (s-1)^2$$

$$= (s-1)^2 + 0 = (s-1)^2$$

$$= s^2 - 2s + 2$$

$$\text{Adj}\left[sI-A\right] = \text{Adj}\begin{bmatrix}a&b\\c&d\end{bmatrix} = \begin{bmatrix}d&-b\\-c&a\end{bmatrix}$$

$$\mathrm{Adj}\,[SI - A] = \mathrm{Adj}\begin{bmatrix} \overset{a}{s-1} & \overset{b}{0} \\ \overset{c}{-1} & \overset{d}{s-1} \end{bmatrix}$$

$$= \begin{bmatrix} \overset{d}{s-1} & \overset{-b}{0} \\ \overset{-c}{1} & \overset{a}{s-1} \end{bmatrix}$$

$$[SI - A]^{-1} = \frac{\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}}{(s-1)^2} \quad \mathrm{Adj}_A = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$\phi(t) = \mathcal{L}^{-1}\,[SI - A]^{-1}$$

$$\boxed{\phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}}$$

state transition Matrix